

Shapes and β -decay in proton rich Ge, Se, Kr and Sr isotopes

P. Sarriguren, E. Moya de Guerra, and A. Escuderos

*Instituto de Estructura de la Materia,
Consejo Superior de Investigaciones Científicas,
Serrano 123, E-28006 Madrid, Spain*

We study ground states and β -decay properties of the proton rich isotope chains Ge, Se, Kr, and Sr. We use a deformed selfconsistent HF+RPA approach with density-dependent effective interactions of Skyrme type. We find that most of the isotopes present two HF minima corresponding to two different shapes. In addition to static quadrupole moments and other ground state band properties, we present results for Gamow-Teller strength distributions, as well as for half-lives and summed strengths. The role of deformation is particularly emphasized.

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Corresponding Author: P. Sarriguren

Postal Address: Instituto de Estructura de la Materia, C.S.I.C., Serrano 123, 28006 Madrid, Spain

e-mail address: emsarri@iem.csic.es

telephone: 34 91 561 6800 (ext. 1126)

fax: 34 91 585 5184

I. INTRODUCTION

The field devoted to the study of exotic nuclei is nowadays one of the most fruitful in Nuclear Physics. Experimental work on nuclei far from stability is providing a wealth of new information [1] that is a challenge to theory. Of prime importance is to test the predictions on unstable nuclei of theoretical models that are trusted for their achievements on stable nuclei.

The interest on exotic nuclei is manifold. On a first step there is the intrinsic appeal to know better and better those regions of the nuclear chart unexplored yet. In addition to that, one has particular interesting problems still open such as the delimitation of the drip lines, the appearance of new phenomena, absent in stable nuclei, from which we can learn new aspects of the nuclear structure, or the decay properties of these radioactive species that are crucial to understand various phases in the stellar evolution [2]. Concerning the last point, nuclear astrophysics is essential to understand the energy generation, the nucleosynthesis, and the abundance of elements in stars. Nuclear astrophysics provides the input (decay properties and cross sections for nuclear reactions of radioactive nuclei), that are needed to model the late phases of the stellar life. Since this input cannot be determined experimentally for the extreme conditions of temperature and density that hold in the interior of the star, reliable theoretical calculations for these processes are absolutely necessary.

Reliable predictions of β -decay strength distributions are also necessary. These are needed for the calculation of beta decay half-lives as well as for all kinds of β -delayed processes like β -delayed particle emission or β -delayed fission. The strength distribution depends on the microscopic structure of the initial and final nuclear wave functions as well as on the interaction which mediates the decay, it can be used to infer information on the nuclear structure or to test different models or approximations. A reliable description of the ground state of the parent nucleus and of the states populated in the daughter nucleus is necessary to obtain a good description of the β -strength distribution, and vice versa, failures to describe such distributions would indicate that an improvement of the theoretical formalism is needed.

Among the microscopic nuclear models designed to describe the properties of the nuclear excitations we can distinguish basically two types of approaches. 1) One is a phenomenological approach where one takes an empirical mean field and assumes a simple separable residual interaction. In this case there is a severe constraint of the method when applied to exotic nuclei, connected to the empirical choice of the potential well and residual force. Since such models are based on parameters locally fitted to the available data on stable nuclei, their extrapolation to exotic nuclei

is at least questionable. 2) The other approach is the selfconsistent approach. Here the consistency of the picture is stressed using an effective interaction, usually a Skyrme interaction, that describes successfully the ground state properties of the nuclei along the periodic table within a Hartree-Fock calculation, and it is also able to describe the excited states from an RPA calculation with residual interactions obtained from the same force. The main difficulty is that the complexity of the calculation increases rapidly with the size of the configuration space and one has to work within limited spaces for nonseparable forces. The practical advantage of approach 1) is that it is possible to calculate nuclear excitations in very large configuration spaces since there is no need to diagonalize matrices whose dimensions grow with the size of the configuration space [3].

One way to combine the good features of both approaches is to construct first the quasiparticle basis selfconsistently from a Hartree-Fock calculation with density-dependent Skyrme forces and pairing correlations in BCS, and then to solve the RPA (or QRPA) equations with a separable residual interaction derived from the same Skyrme force. The separable residual interaction is obtained from the exact particle-hole residual interaction corresponding to the Skyrme force after averaging over the nuclear volume. In this way the consistency (mean field and residual interaction determined from the same effective interaction) and the manageability (the size of the RPA problem does not increase with increasing configuration space), are both exploited. One preserves the reliability of a selfconsistent treatment without losing the capability of using large configuration spaces. This is the framework where our calculations are done.

Our procedure can be viewed as an approximation to the method recently proposed by Van Giai et al. [3]. In Ref. [3] the exact particle-hole residual interaction is first reduced to its Landau-Migdal form and then the RPA matrix is expanded into a finite sum of n separable terms.

In a previous paper [4] we already applied this method to ^{74}Kr with the aim of identifying those elements of the theory to which β -decay may be particularly sensitive. We found that the Gamow Teller (GT) strength distribution was especially sensitive to the nuclear shape and RPA correlations, and we also noted the important role played by the two-body effective interaction, as well as by pairing correlations. Therefore, it was concluded that deformation, pairing and the RPA treatment are ingredients that one cannot avoid in a description of the β -decay in this mass region. In this paper we use this knowledge to calculate the decay properties of a series of isotopes that are being presently measured or are considered as candidates for experimental studies [5]. They are proton rich nuclei in the mass region around $A=70$ ($^{64,66,68,70}\text{Ge}$, $^{68,70,72,74}\text{Se}$, $^{72,74,76,78}\text{Kr}$, $^{76,78,80,82}\text{Sr}$), where deformation including shape coexistence plays an important role.

The study of these isotope chains is worth for several reasons. First of all, this mass region is characterized by a very rich structure giving rise to a large variety of coexistent nuclear shapes. Thus, this region is a good laboratory to test nuclear structure models. In addition to that, the study of various isotope chains opens the possibility to distinguish what is general and what is particular in the behaviour of these nuclei. The systematics also allows one to observe whether the agreement with experiment breaks down as we approach the $N = Z$ isotopes (^{64}Ge , ^{68}Se , ^{72}Kr , ^{76}Sr), that are expected to have some peculiarities because of the $T = 0$ pairing correlations [6]. Another interesting point to discuss is whether the strength distributions of β -decay can be used to extract information on the nuclear shape since clear differences in these distributions could appear depending on the shape of the parent nucleus. It would be interesting to find the most favorable cases for this purpose. Finally, since this mass region is at the border or beyond the scope of the full shell model calculations, predictions for the strength distributions, half-lives, and summed strengths in this mass region obtained from selfconsistent mean field approaches are of especial relevance since they will be probably the most reliable calculations. These results could be used to guide the experimental searches and to compare with other kind of calculations when available.

The paper is organized as follows. In Section 2 we remind briefly the main aspects of our approach and establish our choice for the force and pairing gap parameters. In Section 3 we present the results obtained for the energy distribution of the Gamow Teller strength in those isotopes, as well as integrated quantities that are especially relevant because they can be measured directly such as half-lives or total Gamow Teller strength contained within the Q_{EC} energy window, which in these proton rich nuclei is quite large. Finally in Section 4 we point out some final conclusions and remarks.

II. SUMMARY OF THE THEORY

In this section we summarize briefly the theory involved in the microscopic calculations presented in the next Sections. More details can be found in Refs. [4,7]. Our method consists in a selfconsistent formalism based on a deformed Hartree-Fock (HF) mean field obtained with a Skyrme interaction including pairing correlations in the BCS approximation. The single particle energies, wave functions and occupations are generated from this mean field. We add to the mean field a spin-isospin residual interaction with a coupling strength derived by averaging over the nuclear volume the Landau-Migdal force, obtained from the same energy density functional (and Skyrme interaction) as the HF equation. The residual force is therefore consistent with the mean field. The equations of motion are solved in the proton-neutron quasiparticle random phase approximation (QRPA) [8].

The merits of the density-dependent HF approximation to describe the ground-state properties of both spherical and deformed nuclei are well known [9]. We consider in this paper the force SG2 [10] of Van Giai and Sagawa although we also show results in some instances obtained with the most traditional Skyrme force Sk3 [11]. We use Sk3 in its density dependent two-body version that has better spin-isospin properties than the three-body one [12]. The two forces were designed to fit ground state properties of spherical nuclei and nuclear matter properties but, in addition, the force SG2 gives a good description of Gamow Teller excitations in spherical nuclei [10]. It also provides a good description of spin excitations in deformed nuclei [12].

For the solution of the HF equations we follow the McMaster procedure that is based in the formalism developed in Ref. [13] as described in Ref. [14]. The single-particle wave functions are expanded in terms of the eigenstates of an axially symmetric harmonic oscillator in cylindrical coordinates. We use eleven major shells. The method also includes pairing between like nucleons in the BCS approximation with fixed gap parameters for protons Δ_p , and neutrons Δ_n , which are determined phenomenologically from the odd-even mass differences through a symmetric five term formula involving the experimental binding energies [15].

In the next Section we will discuss shape coexistence. To that end we perform constrained HF calculations with a quadratic quadrupole constraint [16], and analyze the energy surfaces as a function of deformation. The curves are obtained by minimizing the HF energy under the constraint of holding the nuclear deformation fixed. This is carried out over a large range of deformations. When more than one local minimum occurs for the total energy as a function of deformation, shape coexistence results.

For the study of β -decay the relevant residual interactions are the spin-isospin contact forces generating the allowed Gamow Teller transitions. Following Bertsch and Tsai [17] the particle-hole interaction consistent with the HF mean field is obtained as the second derivative of the energy density functional with respect to the one-body density. Neglecting momentum dependent terms, this gives a local interaction that can be put in the Landau-Migdal form [18]

$$V_{ph} = N_0^{-1} \sum_{\ell=0,1} [F_\ell + G_\ell \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + (F'_\ell + G'_\ell \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2] \delta(\mathbf{r}_1 - \mathbf{r}_2). \quad (1)$$

Retaining only the $\ell = 0$ spin-isospin term and averaging the contact interaction over the nuclear volume, we end up with a separable residual ph interaction

$$V_{GT} = 2\chi_{GT} \sum_K (-1)^K \beta_K^+ \beta_{-K}^- \quad (2)$$

in terms of the Gamow Teller operator $\beta_K^\pm = \sigma_K t^\pm$ ($K = 0, \pm 1$). The coupling strength is given by

$$\chi_{GT} = \frac{3}{4\pi R^3} \left(-\frac{1}{2} \right) \left\{ t_0 + \frac{1}{2} k_F^2 (t_1 - t_2) + \frac{1}{6} t_3 \rho^\alpha \right\} = N_0^{-1} \frac{3G'_0}{2\pi R^3} \quad (3)$$

as a function of the Skyrme parameters $t_0, t_1, t_2, t_3, \alpha$, the nuclear radius R , and the Fermi momentum k_F .

The proton-neutron QRPA phonon operator for Gamow Teller excitations in even-even nuclei is written as

$$\Gamma_{\omega_K}^+ = \sum_{pn} [X_{pn}^{\omega_K} \alpha_n^+ \alpha_p^+ - Y_{pn}^{\omega_K} \alpha_n \alpha_p] \quad (4)$$

where $\alpha^+ (\alpha)$ are quasiparticle creation (annihilation) operators, ω_K are the excitation energies, and $X_{pn}^{\omega_K}, Y_{pn}^{\omega_K}$ the forward and backward amplitudes, respectively. The advantages of using a separable residual interaction are well known, the RPA problem can be easily solved no matter how many two-quasiparticle (2qp) configurations are included. The RPA eigenvalues are obtained as the root of a single secular equation and then the corresponding RPA amplitudes can be calculated by performing summations over 2qp states. Explicit expressions of the secular equations that we solve for the $K = 0$ and $K = 1$ Gamow Teller modes are given in Ref. [4].

In the intrinsic frame the Gamow Teller β_K^+ strengths connecting the ground state 0^+ and the excited states $1_{\omega_K}^+$ are obtained as

$$\langle \omega_K | \beta_K^+ | 0 \rangle = \sum_{pn} (u_n v_p X_{pn}^{\omega_K} + v_n u_p Y_{pn}^{\omega_K}) \langle n | \sigma_K | p \rangle \quad (5)$$

where the v 's are the occupation amplitudes ($u^2 = 1 - v^2$). From the RPA equations it is easy to go back to simpler approximations: The Tamm Dancoff approximation (TDA) is recovered by neglecting all the terms involving the backward amplitudes Y . The uncorrelated two-quasiparticle excitations are obtained in the limit of zero residual interaction. The Ikeda sum rule is fulfilled in all of these approximations. For each component $K = 0, \pm 1$, we get

$$\sum_{\omega_K} |\langle \omega_K | \beta_K^- | 0 \rangle|^2 - |\langle \omega_K | \beta_K^+ | 0 \rangle|^2 = N - Z, \quad (6)$$

and summing over K , we obtain $3(N - Z)$ as expected.

In the laboratory frame the transition probability for β^+ decay from the 0^+ to a 1_{ω}^+ state is given by

$$B_{GT}^+(\omega) = \frac{g_A^2}{4\pi} \left\{ \sum_{\omega_0} |\langle \omega_0 | \beta_0^+ | 0 \rangle|^2 \delta(\omega - \omega_0) + 2 \sum_{\omega_1} |\langle \omega_1 | \beta_1^+ | 0 \rangle|^2 \delta(\omega - \omega_1) \right\}. \quad (7)$$

Finally the half-lives are obtained from the B_{GT}^+ strengths within the theoretical energy window Q_{EC} .

III. RESULTS

A. Ground State Properties

The constrained HF method allows one to get the best solution for each value of the mass quadrupole Q_0 . In Figs. 1 to 4 we show the HF energy as a function of the mass quadrupole moment for the two interactions SG2 (solid)

and Sk3 (dashed) in Ge, Se, Kr, and Sr isotopes, respectively. The best HF solution at each Q_0 value is obtained by varying the size and deformation parameters [13] of the deformed harmonic oscillator basis containing 286 states (plus their time reverse). One should note that in these figures the origin of the vertical axis varies for each plot but the unit length (distance between ticks) corresponds always to 1 MeV. Tables 1-4 contain the values of the binding energies obtained in the various cases from which one can deduce the appropriate vertical scale.

As it is seen in Figs. 1-4 in most cases there are two minima close in energy, indicating shape coexistence. Fig. 1 for Ge-isotopes shows that the two solutions are one in the prolate sector and one in the oblate sector in the four isotopes studied. The two forces agree in their predictions on the position of the minima with the only exception of ^{70}Ge , where Sk3 produces a prolate solution at a larger Q_0 value than SG2. The energies of the two minima are quite close (less than 1 MeV apart in all cases), indicating a very favorable case to find shape coexistence in any of these four Ge-isotopes studied. It is remarkable in this case the similarity among the four isotopes. Table 1 contains various ground state properties of these Ge-isotopes for the oblate and prolate solutions of the forces SG2 and Sk3. The first columns contain the pairing gap parameters for neutrons Δ_n and protons Δ_p as derived from the experimental masses [15]. Besides the Skyrme force, they are the only input parameters in our calculation. In the next columns we can find the Fermi energies for neutrons λ_n and protons λ_p , the charge radii r_C , the charge ($Q_{0,p}$) and mass (Q_0) quadrupole moments, the quadrupole deformations β_0 , the values of $\langle J^2 \rangle$, the cranking moments of inertia \mathcal{I}_{cr} , the gyromagnetic ratios g_R , the binding energies E_T , the coupling constant of the residual interaction χ_{GT} , and the Q_{EC} values.

Fig. 2 and Table 2 are the analogous to Fig. 1 and Table 1 for Se-isotopes. In this case, we can see from Fig. 2 the existence again of two solutions in each isotope but now there is a tendency to favor the oblate solution as the ground state. This is true in the four isotopes and with the two forces considered. It is also worth mentioning that with the force SG2 the prolate solution tends to disappear as the number of neutrons increases. When one reaches ^{74}Se , only an oblate and a spherical solution survive. Fig. 3 and Table 3 contain the results for the Kr-isotopes. Here, we still find shape isomerism but the situation now changes considerably from one isotope to another as well as from one force to another. ^{72}Kr exhibits a pronounced oblate ground state shape and a prolate isomer with both forces SG2 and Sk3. The next isotope ^{74}Kr exhibits shape isomerism as well, but its characteristics depend on the force. While SG2 favors an oblate shape, Sk3 favors a prolate one. The situation changes again in ^{76}Kr , where SG2 clearly indicates a spherical ground state while Sk3 predicts an oblate/prolate coexistence. This is also the case in ^{78}Kr . Thus, while the two isomers oblate and prolate survive in all cases with Sk3, the force SG2 predicts an oblate ground state and a

prolate isomer in the $N = Z$ isotope ^{72}Kr . Little by little the oblate ground state collapses into a spherical solution as the number of neutrons increases and the prolate solution finally disappears. Fig. 4 and Table 4 show the results for Sr-isotopes. We can see in this case that the two forces agree in describing ^{82}Sr and ^{80}Sr as spherical but they differ in ^{78}Sr and ^{76}Sr . Sk3 produces a prolate ground state in these two isotopes and a shape isomer which is oblate but almost spherical. On the other hand, SG2 favors a spherical ground state in ^{78}Sr with a prolate isomer and a shape coexistence oblate/prolate in ^{76}Sr .

Numerical comparison with experiment of binding energies shows that the SG2 force gives a small overbinding ($\sim 2\%$), while the Sk3 force gives a small underbinding ($\sim 0.7\%$), systematically in all the nuclei considered. Consistently, we find that the nuclear size, as represented by the r_C values, are systematically somewhat larger with Sk3 than with SG2, both being in good agreement with the available experimental values. This comparison of binding energies and r_C values does not point out to any particular difference between the $N = Z$ and the $N > Z$ even-even isotopes.

Experimental $|\beta_0|$ values, as extracted from $B(E2)$ measurements [20], are also in good agreement with most of our microscopically calculated β_0 values. We would like to recall here that nonzero experimental β values in spherical nuclei correspond to vibrational excitations rather than to the stable deformations calculated here, thus the experimental $|\beta_0|$ value for ^{82}Sr in table 4, corresponds to a vibrational $E2$ transition. The moments of inertia and collective gyromagnetic ratios are given for possible future comparison to theory and experiment. The coupling strengths χ_{GT} are obtained from Eq.(3) using the Sk3 and SG2 Skyrme parameters and $R = 1.2A^{1/3}$ fm. The Q_{EC} values in Tables 1-4 are calculated from our theoretical binding energies,

$$Q_{EC} = m_p - m_n + m_e - (\lambda_n + E_n)_{(N,Z)} + (\lambda_p - E_p)_{(N,Z-2)} \quad (8)$$

B. Gamow Teller Strength Distributions

Before starting to comment the results obtained for the strength distributions, a discussion concerning the residual interaction is in order. As we have already mentioned, the coupling strength of our spin-isospin residual interaction χ_{GT} is obtained from the Skyrme parameters (Eq.(3)) and therefore, the mean field and residual interaction are consistently derived from the same force without any free parameter left. Nevertheless, one could ask how this coupling strength compares with other values previously used in the literature and how well it describes the position of the experimental Gamow Teller resonance (GTR) as obtained from (p, n) reactions. Such comparisons to experiment have been a common method to adjust the coupling strength of the residual spin-isospin force. By this procedure a

value of $\chi_{GT} = 23/A$ MeV was obtained [23] for the coupling strength in Eq.(2). The fit corresponds to the GTR in ^{208}Pb , which is centered at an excitation energy in the daughter nucleus of 15.5 MeV. The value for χ_{GT} was obtained by using the experimental values for the particle and hole energies as explained in Ref. [23] and then it would change if one uses, instead of those experimental energies, the single particle energies as obtained from a selfconsistent mean field calculation as in our case. That means that one should be careful when using this value for the coupling strength because it also implies the use of the experimental energies. As soon as one uses a different set of single particle energies, the fitting procedure should be repeated to extract a new value of χ_{GT} able to reproduce the excitation energy of the GTR within the new framework.

It should also be mentioned that the value of the coupling strength that reproduces the position of the GTR in ^{208}Pb varies if one considers a different mass region. It is known [24] that one needs different values of χ_{GT} to reproduce the GTR in different mass regions. In an attempt to improve the systematics of the dependence of the strength χ_{GT} with the mass number A , more sophisticated dependences than κ/A have been tried [24]. A dependence of the type $\chi_{GT} = \kappa/A^\mu$ has been adjusted to data in Ca, Zr, and Pb. It has been found that $\chi_{GT} = 5.2/A^{0.7}$ is able to reproduce in a reasonable way those data. Again, this parametrization would be dependent on the mean field and single particle energies used.

The value we obtain for the coupling strength is $27/A$ MeV for SG2 and $26/A$ MeV for Sk3, which are quite close and a little bit higher than the value $23/A$, mentioned above. A similar value, $28/A$ MeV, was obtained in Ref. [25] to reproduce the systematics of the energy differences between the GTR and the isobaric analog state observed in (p, n) reactions. With our value we obtain for ^{208}Pb the position of the GT resonance at 19 MeV which is a few MeV larger than experiment. This result was already known for the force SG2. In Ref. [10] it was found, within a TDA calculation with a contact Landau force in ^{208}Pb , that SG2 gives the GTR at 18 MeV while the other force considered in that paper (SG1) produces the peak at 21 MeV. In Ref. [26] the resonance was also found at 19 MeV within an RPA calculation with a Skyrme-Landau interaction. Our results from a separable equivalent force confirm those results. The value of χ_{GT} needed to reproduce in our case the GTR in ^{208}Pb is $\chi_{GT} = 19/A$ MeV.

To illustrate further the comparison of the calculated and experimental positions of the GTR, it is interesting to compare the predictions of our approach using the consistent separable residual interaction in the mass region of our interest here. Unfortunately there is not much experimental information available. One exception is the case of the Fe isotopes, which are probably the most extensively studied nuclei in this mass region because of the interest in astrophysics. The isotopes $^{54,56}\text{Fe}$ have been measured by (p, n) [27] and (n, p) [28] reactions to obtain the GT⁻

and GT^+ strengths, respectively. We can see in Fig. 5 the result of this comparison, where the strength ($L = 0$ forward-angle cross section or GT strength) has been plotted versus the excitation energy of the daughter nucleus. The agreement with the experimental position of the GTR is quite reasonable, especially if one takes into account that the theoretical calculation has no free parameters. It is also clear that one could improve this agreement by reducing a little bit the coupling strength.

Experimental information on (n, p) reactions is also available for $^{70,72}\text{Ge}$ [29], which are much closer to the nuclei of our interest in this paper. Fig. 6 contains this comparison between the experimental $L = 0$ cross sections and the Gamow Teller strength distribution calculated with the force SG2 in RPA and for the two shapes (prolate and oblate in ^{70}Ge , spherical and oblate in ^{72}Ge) that produce HF energy minima. As in the case of the Fe isotopes, the agreement with the experimental excitation energy of the GTR is not bad. The peak in ^{70}Ge is well reproduced while the peak in ^{72}Ge is at the correct energy although experimentally it appears as a broad resonance.

It is not the aim of this paper to fit experimental data on GT strength distributions, but rather to provide results obtained from the consistent value of χ_{GT} —as obtained from the Skyrme force—to avoid playing around with free parameters. In any event, the comparison on Fe and Ge above discussed, shows that the method gives reasonable results. On the other hand, the strength below the Q -window, which is the relevant energy region for β -decay, is in practically all the cases considered here much below the peak of the GTR and therefore not influenced directly by its position within a few MeV.

The Gamow Teller β^+ strength distributions calculated in the selfconsistent HF+RPA scheme with the force SG2 are shown in Figs. 7-10 for the Ge, Se, Kr, and Sr-isotopes, respectively, as a function of the excitation energy of the daughter nucleus. We have folded the calculated GT strengths with $\Gamma = 1$ MeV width Gaussians converting the discrete spectrum into a continuous curve. In these figures, the GT strength of the various isotopes are compared among themselves in a different panel for each nuclear shape. In this way one can appreciate the magnitude of the various strengths on the same scale.

Fig. 7 shows the GT distributions in Ge-isotopes. If one concentrates first on the comparison of the strengths for a given shape, the first thing to notice is that the main peaks of the strength occur at lower energies when one increases the number of neutrons. This is accompanied with a reduction of the strength with increasing neutron number. Now, if we compare the strength distributions of a given isotope obtained from the two shapes, we find that the profiles are in this case quite similar. They are peaked at about the same energy and contain comparable strengths, the oblate ones being a little bit smaller. This is true for the four Ge-isotopes considered and therefore, we can conclude

that the Ge-isotopes are not among the best candidates to look for deformation effects based on the GT strengths distributions.

Fig. 8 contains the GT strength distributions for the Se-isotopes. In this case we also find a clear reduction of the GT strength with the increasing number of neutrons in the oblate and prolate solutions. However, contrary to what happened with the Ge-isotopes, we observe now that the position of the main peaks does not become systematically lower with increasing neutron number, on the contrary, the energy of the main peaks is quite similar for all the isotopes except for the $N = Z$ one that is shifted to higher energies in both cases oblate and prolate. A comparison between the oblate and prolate strength distribution for a given isotope shows that there are not substantial differences between them. The position of the peaks appear at about the same energy and only a slightly smaller strength in the oblate case is worth mentioning. It should also be mentioned that for ^{74}Se the curve shown on the left panel (prolate label) corresponds actually to the spherical solution in Fig. 2 since there is no prolate solution for this nucleus with the SG2 force.

Fig. 9 is the analogous for Kr isotopes. Here one should also take into account that for ^{78}Kr there is a single spherical solution and that the results shown in the panels labelled oblate and prolate correspond actually to this spherical solution. Similarly, the results under the label oblate for ^{76}Kr corresponds actually to the spherical solution in Fig. 3. In this chain of isotopes the changes are more dramatic, especially in what concerns the oblate and prolate differences. The strength again increases as we approach the $N = Z$ isotope and the position of the bumps is also displaced to higher energies. The important new feature here is the strong difference between the calculated strength distributions obtained for the two different shapes. The most remarkable differences are those between the oblate and prolate solutions in ^{74}Kr and between the prolate and spherical solutions in ^{76}Kr . Here we have found firm candidates to study the shapes from their decay properties. Note that the figures corresponding to ^{74}Kr are slightly different from those shown in Ref. [4]. This is simply due to the different values of χ_{GT} used that correspond to a different choice of the nuclear radius R in Eq. (3). In this paper $R = 1.2A^{1/3}\text{fm}$.

The strength distributions in Sr isotopes can be seen in Fig. 10. The trend observed within the prolate solutions is similar to the above mentioned behavior, the strength increases and is shifted to higher energies as we approach $N = Z$. In the right hand side panel, the strength for ^{76}Sr corresponds actually to decay from the oblate solution in Fig. 4. For the true spherical cases ($^{78,80,82}\text{Sr}$), the strengths are noticeably smaller as compared to the deformed shapes. Therefore, this fact can be exploited to study nuclear shapes from β -decay properties as in the previous case for Kr isotopes. We will come back to this point when discussing the strengths summed up to the accessible

experimental window in the next subsection.

In the next set of figures, Figs. 11-14, we compare the GT strength distributions obtained in RPA, TDA, and in the uncorrelated two-quasiparticle case with the force SG2. The general trend seen in these figures is similar to that observed in our previous work on ^{74}Kr [4] and can be summarized as follows: Compared to the uncorrelated two quasiparticle response, RPA produces two types of effects. First, there is a shift of the GT strength to higher energies due to the repulsive character of the spin-isospin residual interaction and second, there is a reduction of the total strength. While the shifting effect is already contained in the TDA description, the quenching effect is not.

Fig. 11 shows this comparison among different approximations for the oblate and prolate shapes of the Ge isotopes. We can see explicitly on the example of this figure the two effects just described. The displacement of the strength to higher excitation energies in TDA and RPA with respect to the uncorrelated case, and the suppression of the strength in RPA. We can also study the dependence on deformation of the GT strength distributions in the uncorrelated basis. If we compare the uncorrelated prolate and oblate distributions (dotted lines) for a given isotope, we arrive to the same conclusion as in the discussion of Fig. 7. There is not a strong dependence on deformation for these Ge isotopes, although now some differences become more apparent. For example there is a first bump at very small energies in all the oblate cases that is almost suppressed in the prolate ones. These bumps are redistributed by the action of the residual force and a much smoother strength distribution is found in RPA. Nevertheless, there is still a small bump, reminiscent of the peak in the uncorrelated case, that appears at small energies in the oblate cases and that, as we shall see later on, plays an important role because it is a signature of an oblate shape in the parent nucleus that can be identified by measuring the GT strength at low excitation energies below the Q_{EC} window. Thus, although the RPA strength distributions are smooth out in comparison to the more sensitive uncorrelated distributions, there are still traces of that sensitivity which can be exploited to probe the shape of the nucleus.

The effect of residual interaction and RPA correlations for Se, Kr, and Sr isotopes are shown in Figs 12,13, and 14, respectively. The case of Se is very similar to that of Ge, there are no strong deformation effects. On the contrary, for Kr and Sr isotopes the dependence on deformation of the uncorrelated strength distributions is huge and this is the origin of the deformation dependence in RPA discussed earlier for these isotopes.

Figs. 15-18 show the dependence of the HF+RPA Gamow Teller strength distributions to the Skyrme interaction used in the calculations. The results are for SG2 (solid line) and Sk3 (dashed line) in all the isotopes considered. In Fig. 15 for Ge isotopes we can see that there is almost no difference in going from one interaction to another and thus, the conclusions have a general validity. Fig. 16 for Se isotopes shows the same characteristics. The profiles

obtained with both interactions are quite similar. The larger discrepancies occur in the prolate solutions of ^{74}Se and ^{72}Se , but this is mainly due to the different minima obtained for these two nuclei with the two interactions (see Fig. 2), while Sk3 produces well deformed prolate solutions, SG2 has an almost spherical solution for these two isotopes. Fig. 17 shows the results for Kr isotopes. Here again, the strength distributions obtained with the two interactions are quite similar in the cases where the HF solutions appear at about the same deformation. On the other hand, when the HF solutions occurred at different deformations in Sk3 and SG2, the strength distributions obtained from those solutions are also quite different. This is clearly the case in ^{78}Kr , where Sk3 has two solutions oblate and prolate while SG2 has a single spherical solution. This is also true to a lesser extent in the oblate solutions of ^{76}Kr and ^{74}Kr and in the prolate solution of ^{72}Kr that occur at different deformations. On the other hand, the rest of cases have very similar strength distributions and they also have very similar deformations in the HF solutions (see Fig. 3). In Fig. 18 we can see the results for Sr isotopes. The profiles of the strength distributions are in this case practically the same in accordance with the situation in Fig. 4, where the HF solutions with the two interactions occur at the same deformations.

C. Half-lives and Summed Strengths

In this subsection we present the results obtained for other quantities of interest such as the half-lives or the GT strengths summed up to the Q_{EC} window (\sum_{EC}). One of the points to discuss in this context is whether there are substantial differences in these quantities depending on the shape of the parent nucleus. In the affirmative case, this implies that experimental data on β -decay can be taken as a signature of the nuclear shape. Thus, it is instructive to see the predictions for the half-lives and summed strengths corresponding to the different stable shapes.

The total half-life $T_{1/2}$ for allowed β decay from the ground state of the parent nucleus is given by summing over all the final states involved in the process

$$T_{1/2}^{-1} = \frac{\kappa^2}{D} \sum_{\omega} f(Z, \omega) |\langle 1_{\omega}^{+} \| \beta^{+} \| 0^{+} \rangle|^2 \quad (9)$$

The Fermi integrals $f(Z, \omega)$ are taken from Ref. [30]. We use $D = 6200$ s and include effective factors

$$\kappa^2 = \left[(g_A/g_V)_{eff} \right]^2 = \left[0.77 (g_A/g_V)_{free} \right]^2 = 0.90 \quad (10)$$

to take into account in an effective way the quenching of the GT coupling constant g_A in the nuclear medium. Note that the value of the standard quenching factor (0.77) used here [31] is a little bit different than the value (0.7) used in Ref. [4], and consequently the values of $T_{1/2}$ have changed in accordance.

Tables 5-7 show the results obtained from bare $2qp$, TDA, and RPA calculations for the GT strength summed up to an energy cut of 30 MeV (Table 5), for the GT strength summed up to excitation energies below Q_{EC} (Table 6), and for the total β^+/EC half-life (Table 7). The cut of 30 MeV corresponds to the excitation energy for which the Ikeda sum rule is fulfilled up to a few per thousand. Results are shown for the two Skyrme forces Sk3 and SG2, as well as for the different shapes oblate (o), prolate (p), or spherical (s), where the minima occur for each isotope.

One can see in Table 5 that the summed GT strengths up to the energy cut are conserved in going from $2qp$ to TDA calculations, but this is no longer true in RPA where the strengths are reduced. The energy weighted sums, not shown here, have the opposite behavior, the two quasiparticle values are conserved by RPA while TDA produces larger energy weighted sums (see also Ref. [4]). Focussing on the RPA total strengths, one can see on the table that the prolate shape tends to give a larger total strength, this is more noticeable in Kr and Sr isotopes when the other equilibrium shape is spherical. In comparing RPA results for Sk3 and SG2 one can see that the GT summed strengths are practically equal when the predicted shapes have similar β_0 values. This implies that there is a strong correlation between the nuclear shape and the total GT strength within the Q_{EC} window (\sum_{EC}).

As a matter of consistency the GT excitations and Q_{EC} values from which we obtain the summed strengths \sum_{EC} in Table 6 and the half-lives $T_{1/2}$ in Table 7 have been calculated in the parent nucleus for each force, shape and approach. In particular for the Q_{EC} value in RPA and TDA approximations the energy of the lowest two-quasiparticle state is replaced by the energy ω of the lowest RPA or TDA state, respectively. In most cases however the Q_{EC} values in various approaches differ at most by a few percent.

In tables 6-7, we do not include stable nuclei (^{70}Ge , ^{74}Se , ^{78}Kr) or nuclei near to stability with very small Q_{EC} values (^{68}Ge , ^{72}Se , ^{82}Sr , having $Q_{EC} < 0.4$ MeV, see Tables 1-4). Quantities depending on the value of the cut Q_{EC} , such as those in Tables 5-6, are extremely sensitive to the cut when Q_{EC} is very small. In this case only very few low energy excitations contribute to \sum_{EC} and $T_{1/2}$ and a small change in the Q_{EC} value may lead to large variations in these quantities, specially in the half-lives that can change by orders of magnitude. Generally, when Q_{EC} is large enough small changes in Q_{EC} are followed by small changes in $T_{1/2}$. This is especially true in the deformed case, where the excitation energies are very much fragmented and appear in an almost continuous distribution. In contrast, in the spherical case the existence of large strengths at well located excitation energies can make the half-lives much more dependent on fine details of the calculations.

Note that while the half-lives in Table 7 contain already the expected quenching factor (see Eqs. (9,10)), the strengths in Table 6 are in units of $[g_A^2/4\pi]$, and therefore a reduction of about a 50% is expected in these strengths

before comparison to experiment is made due to the effective g_A value.

A common feature to both tables is that the calculated half-lives $T_{1/2}$ (summed strengths \sum_{EC}) increase (decrease) in going from $2qp$ to TDA to RPA. One finds variation factors of the order of ten between $2qp$ and RPA calculations in \sum_{EC} and $T_{1/2}$. TDA is in all cases much closer to RPA than to $2qp$ calculations but still observable differences appear in some cases. Therefore, from these calculations one concludes that in order to achieve a reliable description of β -decay properties, an RPA calculation must be performed.

Comparison to the experimental half-lives in Table 7 shows that the RPA results agree in general with experiment. The only exception are the $N = Z$ isotopes of Ge (^{64}Ge) and Se (^{68}Se), where we overestimate the half-lives by a factor between 2.5 and 5. Even in the cases not included in the table because of their small Q_{EC} value (^{68}Ge , ^{72}Se , ^{82}Sr), we obtain half-lives of the order of days as in experiment.

In a more detailed analysis we can see that the RPA summed strengths within the Q_{EC} window (half-lives) in the Ge isotopes are smaller (larger) for the prolate shapes than for the oblate ones. This is due to the small peak that appears in the distribution of the strength in the oblate cases, absent in the prolate ones (see Figs. 7,11,15). This could lead to an observable effect in $^{64,66}\text{Ge}$. Although the total strength (sums up to 30 MeV) are in most cases larger for the prolate shapes, the opposite happens in the sums cut at Q_{EC} . It is also important to mention that SG2 and Sk3 agree in their predictions for the summed strengths. The results obtained for the Se isotopes do not show any remarkable pattern and then the Se isotopes are not good candidates to look for sizeable effects on the GT strengths due to deformation.

Special attention deserve the cases of Kr and Sr isotopes. The summed GT strengths up to the Q_{EC} window are not conclusive to distinguish between oblate or prolate shapes in $^{72,76}\text{Kr}$. The situation is different in ^{74}Kr . In this nucleus one obtains the same strength with the two forces in the oblate case, strength which is much smaller than the strength obtained in the calculation with the prolate shape. This fact makes ^{74}Kr a suitable candidate to measure its GT strength and from this measurement to infer the ground state shape. In a similar way, \sum_{EC} are about the same in the case of ^{76}Sr , where a coexistence between oblate and prolate shapes is predicted. On the other hand, in the other two cases $^{78,80}\text{Sr}$ where a prolate and spherical shape coexistence appears, \sum_{EC} calculated in the prolate shape is clearly larger than the corresponding strength calculated from the spherical shape. Therefore, these two nuclei are again very interesting cases to look for these deformation effects on the GT strengths.

D. The particle-particle residual interaction

It has often been claimed (see for instance Ref. [32] and refs. therein) that for a complete description of the β^+ and $\beta\beta$ strengths, the inclusion of the particle-particle (pp) residual interaction is required. Therefore the question may arise as to why this interaction was not included in the present work. The usual way to include this force is in terms of a separable force with a free coupling constant κ_{pp} , which is fitted to the phenomenology. Since the peak of the GTR is almost insensitive to the pp force, κ_{pp} is usually adjusted to reproduce the half-lives.

One of the features of the pp force is that, being an attractive force, the GT strength is pushed down to lower energies with increasing values of κ_{pp} . If κ_{pp} is strong enough it may happen that the RPA collapses, because the condition that the ground state be stable against the corresponding mode is not fulfilled. Inconsistencies between mean field and residual interactions are a source of problems, particularly when discussing single β or double β decays. There is work in progress to include the pp residual force in a consistent way, starting from Hartree-Fock-Bogoliubov calculations, where proton-neutron pairing is included. Until this project is carried out, we have adopted in this work the value $\kappa_{pp} = 0$, which is consistent with the HF+BCS energy density functional without proton-neutron pairing used here. Furthermore, it has been shown [33] that for small values of κ_{pp} far from the collapse, the half-lives are nearly independent of the pp force.

Nevertheless, just as an illustration we show in this section the effect of the inclusion of a pp force on the GT strength distributions and half-lives. For that purpose, we introduce in our formalism a separable residual pp force in the same way as it was done in Ref. [34]. Using separable forces, the QRPA equation for the separable particle-hole and particle-particle forces can be reduced to an algebraic equation, which is now of fourth order by the inclusion of the pp force. For the solution of the algebraic equation we follow Ref. [34].

We can see in Fig. 19 the effect of the residual pp interaction on the GT strength distributions by changing the κ_{pp} value. The figure corresponds to the prolate and oblate solutions of the nucleus ^{70}Se . It is an RPA calculation with the force SG2. As it can be seen in this figure, the attractive character of the pp force makes the strength to be slightly shifted to lower excitation energies, but the position of the GTR is hardly modified by the inclusion of the pp force. We can also see in Table 8 the total GT strength summed up to 30 MeV, as well as the sums up to Q_{EC} (\sum_{EC}), and the half-lives. The total GT strength is reduced as the value of κ_{pp} increases, but \sum_{EC} increases because of the concentration of the strength at lower energies. As a consequence the half-lives are reduced with increasing κ_{pp} . This is so until the collapse of the RPA takes place, which for the case discussed here occurs at about $\kappa_{pp} = 0.10$ MeV.

We also note that if we would fix κ_{pp} to fit the experimental value of the half-life, we would need $\kappa_{pp} = 0.02$ MeV in the prolate case and $\kappa_{pp} = 0.08$ MeV in the oblate case, which is close to the collapse.

IV. SUMMARY AND FINAL REMARKS

We have investigated shape isomerism and β -decay in several Ge, Se, Kr, and Sr isotopes on the basis of the selfconsistent HF+RPA framework with Skyrme forces. This is a well founded method that has been successfully used to describe quite diverse properties of stable spherical and deformed nuclei through the nuclear chart. It has the appealing feature of treating the excitations and the ground state in a selfconsistent framework with no free parameters. This feature is particularly desirable for nuclei far from stability, where extrapolations of methods based on local fits are more doubtful. We took here the challenge to test the predictions of this method on the above mentioned chains including unstable isotopes. Very reasonable agreement with both ground state and β -decay properties is obtained.

Compared to the uncorrelated two quasiparticle response, RPA shifts the GT strength to higher energies and reduces the total strength. While the shifting effect is already contained in the TDA description, the quenching effect is not. This effect produces half-lives that are much larger in RPA than in the bare 2qp approach. Inclusion of RPA correlations are clearly necessary for comparison to experiment.

We have found shape isomerism in most of the isotopes studied. The RPA Gamow Teller β^+ strength distributions depend on the shape (prolate, spherical or oblate) of the parent nucleus. It is important to notice that these results do not depend much on which effective Skyrme interaction (Sk3 or SG2) is used.

The different nuclear shapes lead in some cases to sizeable differences in the observable range of β^+ -decay. We find that ^{74}Kr , ^{78}Sr and ^{80}Sr are particularly interesting cases to look experimentally for shape effects in β^+ -decay.

For even-even nuclei, neutron-proton $T = 0$ and $T = 1$ pairing is known to be important when $N = Z$ [6]. Since our theoretical treatment does not explicitly include neutron-proton ($n - p$) pairing, we may expect larger deviations between theory and experiment in the $N = Z$ isotopes. The comparison in tables 1-4 of bulk properties like binding energies and r.m.s. radii shows that the agreement between theory and experiment is as good for the $N = Z$ as for the $N = Z + 2, Z + 4, Z + 6$ isotopes. This allows us to conclude that the effect of $n - p$ pairing correlations in the binding energy is roughly taken into account by the use of the phenomenological gap parameters Δ_p, Δ_n . This could be expected from HFB theory where the total gap satisfies

$$|\Delta_p|^2 = |\Delta_{pp}|^2 + |\Delta_{pn}^{T=1}|^2 + |\Delta_{pn}^{T=0}|^2$$

and similarly for neutrons.

β^+ -decay strength functions and half-lives of $N = Z$ nuclei are expected to be more sensitive to the explicit inclusion of $n - p$ pairing in the microscopic calculations. Indeed, a look at our RPA results in table 7 shows that for the $N = Z$ isotopes of Ge and Se, the experimental half-lives are overestimated by a factor of 3 to 5, depending on the interaction and shape, while fair agreement with experimental half-lives is obtained for the $N > Z$ isotopes. Interestingly enough our RPA results for the $N = Z$ isotopes of Kr and Sr are in good agreement with experiment. It will therefore be interesting to see how the inclusion of $n - p$ pairing in our microscopic calculation affects our present results. It will also be interesting to compare our results with future data on β^+ -strengths.

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Figure captions

Figure 1. Total energy of the Ge isotopes $^{64,66,68,70}\text{Ge}$ as a function of the mass quadrupole moment Q_0 . The results correspond to a constrained HF+BCS calculation with the Skyrme interaction SG2 (solid line) and Sk3 (dashed line). The distance between two ticks in the vertical axes is always 1 MeV but the origin is different for each curve.

Figure 2. Same as in Fig. 1 for the Se isotopes $^{68,70,72,74}\text{Se}$.

Figure 3. Same as in Fig. 1 for the Kr isotopes $^{72,74,76,78}\text{Kr}$.

Figure 4. Same as in Fig. 1 for the Sr isotopes $^{76,78,80,82}\text{Sr}$.

Figure 5. (p, n) and (n, p) $L = 0$ cross sections in $^{54,56}\text{Fe}$ compared to theoretical GT strength distributions obtained with the force SG2 in RPA. Experimental data for (p, n) and (n, p) reactions are from [27] and [28], respectively.

Figure 6. (n, p) $L = 0$ cross sections in $^{70,72}\text{Ge}$ [29] compared with the RPA theoretical GT strength distributions obtained from SG2.

Figure 7. Comparison of the Gamow Teller strength distribution $[g_A^2/4\pi]$ in the Ge isotopes $^{64,66,68,70}\text{Ge}$. The results are for the force SG2 in RPA.

Figure 8. Same as in Fig. 7 for the Se isotopes $^{68,70,72,74}\text{Se}$.

Figure 9. Same as in Fig. 7 for the Kr isotopes $^{72,74,76,78}\text{Kr}$.

Figure 10. Same as in Fig. 7 for the Sr isotopes $^{76,78,80,82}\text{Sr}$.

Figure 11. Comparison of RPA (solid line), TDA (dashed line), and bare two-quasiparticle (dotted line) Gamow Teller strength distributions $[g_A^2/4\pi]$ in the Ge isotopes $^{64,66,68,70}\text{Ge}$. The results correspond to the force SG2.

Figure 12. Same as in Fig. 11 for the Se isotopes $^{68,70,72,74}\text{Se}$.

Figure 13. Same as in Fig. 11 for the Kr isotopes $^{72,74,76,78}\text{Kr}$.

Figure 14. Same as in Fig. 11 for the Sr isotopes $^{76,78,80,82}\text{Sr}$.

Figure 15. Gamow Teller strength distributions $[g_A^2/4\pi]$ in the Ge isotopes $^{64,66,68,70}\text{Ge}$ as a function of the excitation energy of the daughter nucleus. The results correspond to the forces SG2 (solid line) and Sk3 (dashed line) in RPA.

Figure 16. Same as in Fig. 15 for the Se isotopes $^{68,70,72,74}\text{Se}$.

Figure 17. Same as in Fig. 15 for the Kr isotopes $^{72,74,76,78}\text{Kr}$.

Figure 18. Same as in Fig. 15 for the Sr isotopes $^{76,78,80,82}\text{Sr}$.

Figure 19. GT strength distributions in ^{70}Se calculated in RPA with the force SG2 for various values of the coupling strength κ_{pp} of the particle-particle force.

Table 1. Pairing gap parameters for neutrons and protons Δ_n, Δ_p [MeV]; Fermi energies for neutrons and protons λ_n, λ_p [MeV]; charge radii r_C [fm]; charge $Q_{0,p}$ and mass Q_0 quadrupole moments [fm²]; values of the deformation parameters $\beta_0, \langle J^2 \rangle$, cranking moments of inertia \mathcal{I}_{cr} [MeV⁻¹], gyromagnetic ratios g_R , binding energies E_T [MeV], coupling strengths of the residual spin-isospin interaction χ_{GT} [MeV], and Q_{EC} [MeV] values for the chain of Ge isotopes ^{64,66,68,70}Ge. Experimental values for $r_C, \beta_0, E_T, Q_{EC}$ are from Ref. [19,20,15], respectively.

			Δ_n	Δ_p	λ_n	λ_p	r_C	$Q_{0,p}$	Q_0	β_0	$\langle J^2 \rangle$	\mathcal{I}_{cr}	g_R	E_T	χ_{GT}	Q_{EC}	
							th.	exp.		th.	exp.			th.	exp.	th.	exp.
⁶⁴ Ge	oblate	SG2	2.10	1.80											-545.9		4.41
		Sk3			-13.43	-3.42	4.01	-122	-238	-0.19	14.8	2.0	0.57	-558.4	0.43	4.3	
	prolate	Sk3			-13.09	-3.09	4.03	-111	-217	-0.17	12.4	1.7	0.56	-541.6	0.42	4.2	
		SG2			-13.32	-3.28	4.00	121	236	0.19	12.4	1.8	0.57	-558.4	0.43	4.1	
		Sk3			-13.05	-3.02	4.03	128	249	0.20	13.5	1.9	0.56	-542.2	0.42	3.9	
⁶⁶ Ge	oblate	SG2	1.80	1.60						± 0.17					-569.3		2.10
		Sk3			-12.35	-4.60	4.02	-134	-272	-0.21	20.2	3.1	0.50	-582.3	0.41	2.1	
	prolate	Sk3			-11.99	-4.48	4.06	-132	-270	-0.21	19.6	2.9	0.47	-564.8	0.40	1.9	
		SG2			-12.21	-4.40	4.01	125	251	0.20	15.1	2.5	0.52	-581.8	0.41	2.1	
		Sk3			-11.83	-4.32	4.05	135	274	0.21	169.	2.6	0.51	-564.8	0.40	1.7	
⁶⁸ Ge	oblate	SG2	1.90	1.60						± 0.20					-590.8		0.11
		Sk3			-11.39	-5.69	4.04	-131	-274	-0.20	18.8	2.8	0.53	-604.8	0.40	0.1	
	prolate	Sk3			-11.02	-5.76	4.08	-135	-285	-0.21	20.0	2.9	0.46	-586.7	0.39	-0.5	
		SG2			-11.36	-5.48	4.02	116	237	0.18	13.0	2.1	0.56	-604.1	0.40	0.3	
		Sk3			-10.89	-5.57	4.07	136	285	0.21	17.2	2.6	0.50	-586.4	0.39	-0.6	
⁷⁰ Ge	oblate	SG2	1.90	1.60			4.05			± 0.22					-610.5		–
		Sk3			-10.70	-6.65	4.04	-111	-236	-0.17	13.6	2.2	0.55	-625.6	0.39	-1.3	
	prolate	Sk3			-10.27	-6.84	4.09	-118	-255	-0.18	15.4	2.3	0.48	-606.8	0.38	-2.1	
		SG2			-10.83	-6.44	4.03	78	157	0.11	5.6	1.0	0.68	-624.8	0.39	-1.1	
		Sk3			-10.18	-6.77	4.09	144	315	0.22	18.8	2.8	0.46	-606.1	0.38	-2.0	

Table 2. Same as in Table 1 for the Se isotopes $^{68,70,72,74}\text{Se}$.

		Δ_n	Δ_p	λ_n	λ_p	r_C	$Q_{0,p}$	Q_0	β_0	$\langle J^2 \rangle$	\mathcal{I}_{cr}	g_R	E_T	χ_{GT}	Q_{EC}	
									th.	exp.			th.	exp.	th.	exp.
^{68}Se		2.20	1.80										-576.4			4.70
	oblate	SG2		-13.61	-3.15	4.10	-153	-299	-0.22		21.1	3.0	0.57	-590.3	0.40	4.1
		Sk3		-13.43	-2.98	4.13	-154	-302	-0.22		21.3	3.0	0.57	-572.6	0.39	4.2
	prolate	SG2		-13.49	-2.98	4.08	134	262	0.19		14.9	2.2	0.59	-589.2	0.40	4.4
		Sk3		-13.30	-2.82	4.12	145	283	0.20		16.6	2.3	0.58	-572.0	0.39	4.2
^{70}Se		1.80	1.80						± 0.29				-600.3			2.40
	oblate	SG2		-12.36	-4.26	4.11	-155	-311	-0.22		22.9	3.4	0.50	-614.1	0.39	2.2
		Sk3		-12.14	-4.24	4.15	-161	-328	-0.23		25.0	3.7	0.46	-596.1	0.38	2.1
	prolate	SG2		-12.36	-4.11	4.09	126	251	0.18		14.5	2.3	0.52	-612.9	0.39	2.6
		Sk3		-12.04	-4.11	4.14	153	310	0.21		20.1	2.9	0.48	-595.1	0.38	2.0
^{72}Se		1.80	1.80						± 0.21				-622.4			0.34
	oblate	SG2		-11.68	-5.28	4.11	-129	-264	-0.18		16.2	2.6	0.51	-636.8	0.38	0.7
		Sk3		-11.36	-5.43	4.16	-156	-326	-0.21		23.1	3.4	0.45	-618.3	0.37	0.3
	prolate	SG2		-11.89	-5.22	4.09	76	148	0.10		4.9	0.9	0.63	-635.6	0.38	1.2
		Sk3		-11.44	-5.56	4.20	248	526	0.34		42.0	5.7	0.38	-617.5	0.37	0.4
^{74}Se		1.80	1.80						± 0.30				-642.9			-
	oblate	SG2		-10.97	-6.33	4.13	-131	-277	-0.18		16.6	2.7	0.45	-658.6	0.37	-1.2
		Sk3		-10.66	-6.52	4.17	-134	-285	-0.18		17.5	2.8	0.42	-639.5	0.36	-2.0
	spherical	SG2		-11.26	-6.29	4.10	0	0	0		0	0	-	-658.2	0.37	-1.0
	prolate	Sk3		-10.55	-6.57	4.20	226	493	0.31		36.3	5.0	0.37	-638.4	0.36	-2.0

Table 3. Same as in Table 1 for the Kr isotopes $^{72,74,76,78}\text{Kr}$. Experimental values for r_C are from Ref. [21].

		Δ_n	Δ_p	λ_n	λ_p	r_C		$Q_{0,p}$	Q_0	β_0		$\langle J^2 \rangle$	\mathcal{I}_{cr}	g_R	E_T		χ_{GT}	Q_{EC}	
						th.	exp.			th.	exp.				th.	exp.		th.	exp.
^{72}Kr		1.50	1.50				4.163									-607.1			5.04
	oblate	SG2		-13.24	-2.57	4.17		-189	-367	-0.25		31.8	5.1	0.52	-618.5		0.38	4.9	
		Sk3		-13.26	-2.64	4.22		-210	-412	-0.27		39.3	6.1	0.52	-601.2		0.37	4.9	
	prolate	SG2		-13.44	-2.65	4.13		108	210	0.14		11.9	2.3	0.51	-617.2		0.38	5.2	
		Sk3		-13.36	-2.73	4.22		237	465	0.30		38.6	5.8	0.51	-599.9		0.37	5.2	
^{74}Kr		1.50	1.50				4.187				± 0.39					-631.3			3.14
	oblate	SG2		-12.74	-3.90	4.15		-112	-222	-0.15		13.4	2.4	0.50	-643.9		0.37	3.2	
		Sk3		-12.30	-3.80	4.23		-204	-412	-0.26		21.5	3.6	0.49	-625.4		0.36	3.0	
	prolate	SG2		-12.75	-4.14	4.23		307	623	0.39		60.2	9.1	0.46	-642.9		0.37	3.3	
		Sk3		-12.59	-4.29	4.27		311	633	0.39		60.3	8.9	0.46	-625.9		0.36	2.7	
^{76}Kr		1.60	1.70				4.202				± 0.41					-654.2			1.31
	spherical	SG2		-12.23	-5.18	4.15		0	0	0		0	0	-	-668.5		0.36	1.4	
	oblate	Sk3		-11.76	-5.01	4.21		-142	-294	-0.18		18.4	3.0	0.40	-649.3		0.35	0.9	
	prolate	SG2		-11.76	-5.09	4.25		299	623	0.37		54.9	4.9	0.44	-666.7		0.36	1.1	
		Sk3		-11.36	-5.37	4.29		302	634	0.37		56.3	7.7	0.44	-649.2		0.35	0.6	
^{78}Kr		1.70	1.80				4.204				± 0.34					-675.6			-
	spherical	SG2		-11.73	-6.18	4.16		0	0	0		0	0	-	-691.3		0.35	-0.7	
	oblate	Sk3		-10.99	-6.17	4.23		-142	-301	-0.18		17.8	2.9	0.38	-671.7		0.34	-0.9	
	prolate	Sk3		-10.81	-6.22	4.28		256	546	0.31		41.1	5.7	0.44	-670.6		0.34	-1.1	

Table 4. Same as in Table 1 for the Sr isotopes $^{76,78,80,82}\text{Sr}$. Experimental values for r_C are from Ref. [22].

			Δ_n	Δ_p	λ_n	λ_p	r_C	$Q_{0,p}$	Q_0	β_0	$\langle J^2 \rangle$	\mathcal{I}_{cr}	g_R	E_T	χ_{GT}	Q_{EC}	
							th.	exp.		th.	exp.			th.	exp.	th.	exp.
^{76}Sr	oblate	SG2	1.50	1.50										-638.1		6.10	
		Sk3			-13.87	-2.66	4.20		-94	-183	-0.11	9.4	1.9	0.51	-649.9	0.36	5.2
	prolate	Sk3			-13.77	-2.59	4.23		-109	-213	-0.13	11.3	2.0	0.51	-630.6	0.35	5.7
		SG2			-13.78	-2.67	4.30		359	702	0.42	69.7	10.3	0.51	-649.5	0.36	5.2
		Sk3			-13.77	-2.71	4.34		358	703	0.41	69.5	10.0	0.51	-632.8	0.35	4.9
^{78}Sr	spherical	SG2	1.30	1.30			4.217			± 0.43				-663.0		3.76	
		Sk3			-13.19	-4.02	4.19		0	0	0	0	0	-	-676.2	0.35	3.9
	prolate	Sk3			-12.98	-3.90	4.23		0	0	0	0	0	-	-656.0	0.34	3.2
		SG2			-12.46	-3.62	4.31		364	732	0.42	74.5	11.3	0.54	-674.3	0.35	2.7
		Sk3			-12.25	-3.83	4.35		369	746	0.42	75.7	11.0	0.54	-657.5	0.34	3.7
^{80}Sr	spherical	SG2	1.60	1.60			4.217			± 0.38				-686.3		1.87	
		Sk3			-12.19	-4.92	4.21		0	0	0	0	0	-	-701.4	0.34	1.5
	prolate	Sk3			-12.22	-4.99	4.25		0	0	0	0	0	-	-681.4	0.33	0.8
		SG2			-11.88	-4.51	4.31		336	686	0.38	60.4	8.7	0.53	-698.1	0.34	1.4
		Sk3			-11.46	-4.77	4.36		340	699	0.39	61.0	8.5	0.53	-680.6	0.33	1.7
^{82}Sr	spherical	SG2	1.70	1.80			4.209			± 0.29				-708.1		0.18	
		Sk3			-11.60	-5.86	4.23		0	0	0	0	0	-	-724.8	0.33	0.1
		Sk3			-11.57	-6.03	4.27		0	0	0	0	0	-	-705.2	0.32	-0.3

Table 5. Results from bare $2qp$, TDA, and RPA calculations for the Gamow Teller strength in units of $[g_A^2/4\pi]$ summed up to $E_{cut} = 30$ MeV. The results correspond to the two Skyrme forces SG2 and Sk3, as well as for the different shapes oblate (o), prolate (p), or spherical (s), where the minima occur for each isotope.

		SG2			Sk3				
		2qp	TDA	RPA			2qp	TDA	RPA
^{64}Ge	(o)	14.2	14.2	9.8	(o)	14.3	14.3	9.9	
	(p)	14.5	14.5	10.0	(p)	14.0	14.0	9.8	
^{66}Ge	(o)	9.7	9.7	6.2	(o)	9.7	9.7	6.3	
	(p)	10.6	10.6	6.7	(p)	10.1	10.1	6.5	
^{68}Ge	(o)	7.4	7.4	4.3	(o)	7.4	7.4	4.3	
	(p)	8.4	8.4	4.7	(p)	8.0	8.0	4.7	
^{70}Ge	(o)	5.5	5.5	2.9	(o)	5.6	5.6	2.9	
	(p)	6.6	6.6	3.2	(p)	6.7	6.7	3.5	
^{68}Se	(o)	12.5	12.5	9.1	(o)	12.6	12.6	9.1	
	(p)	13.5	13.5	9.6	(p)	13.2	13.2	9.5	
^{70}Se	(o)	8.8	8.8	5.8	(o)	9.0	9.0	6.0	
	(p)	10.0	10.0	6.3	(p)	9.8	9.8	6.4	
^{72}Se	(o)	6.6	6.6	3.9	(o)	7.0	7.0	4.2	
	(p)	7.7	7.7	4.2	(p)	9.7	9.7	5.6	
^{74}Se	(o)	4.9	4.9	2.6	(o)	5.1	5.1	2.8	
	(s)	4.7	4.7	2.4	(p)	8.1	8.1	4.3	
^{72}Kr	(o)	10.9	10.9	8.0	(o)	11.3	11.3	8.3	
	(p)	11.5	11.5	8.1	(p)	12.9	12.9	9.2	
^{74}Kr	(o)	6.8	6.8	4.6	(o)	9.2	9.2	6.1	
	(p)	11.3	11.3	7.3	(p)	11.5	11.5	7.5	
^{76}Kr	(s)	4.7	4.7	2.8	(o)	6.2	6.2	3.7	
	(p)	10.2	10.2	5.9	(p)	10.3	10.3	6.1	
^{78}Kr	(s)	3.5	3.5	1.9	(o)	5.3	5.3	2.9	
					(p)	9.0	9.0	4.8	
^{76}Sr	(o)	8.7	8.7	6.6	(o)	9.4	9.4	7.0	
	(p)	14.2	14.2	10.2	(p)	14.4	14.4	10.4	
^{78}Sr	(s)	4.0	4.0	2.8	(s)	5.1	5.1	3.4	
	(p)	12.1	12.1	7.8	(p)	12.4	12.3	8.1	
^{80}Sr	(s)	3.7	3.7	2.4	(s)	4.8	4.8	2.9	
	(p)	11.4	11.4	6.5	(p)	11.5	11.5	6.8	
^{82}Sr	(s)	3.7	3.7	2.1	(s)	4.5	4.5	2.5	

Table 6. Same as Table 5 for the summed GT strength contained in the Q_{EC} window (\sum_{EC}).

		2qp	SG2 TDA	RPA			2qp	Sk3 TDA	RPA
^{64}Ge	(o)	10.7	1.2	1.0	(o)	9.5	1.2	1.0	
	(p)	11.6	0.9	0.8	(p)	10.3	0.8	0.7	
^{66}Ge	(o)	4.0	0.7	0.5	(o)	2.4	0.7	0.5	
	(p)	6.4	0.5	0.4	(p)	1.9	0.5	0.3	
^{68}Se	(o)	8.7	1.7	1.5	(o)	8.2	1.8	1.5	
	(p)	10.1	2.0	1.8	(p)	9.2	1.6	1.4	
^{70}Se	(o)	3.9	0.9	0.7	(o)	2.9	1.1	0.8	
	(p)	6.2	1.5	1.3	(p)	6.0	1.6	0.4	
^{72}Kr	(o)	7.6	3.4	2.7	(o)	7.0	3.0	2.3	
	(p)	9.3	5.0	4.1	(p)	10.1	2.8	2.4	
^{74}Kr	(o)	3.5	1.6	1.1	(o)	3.6	1.6	1.1	
	(p)	7.3	2.3	1.9	(p)	7.3	1.7	1.5	
^{76}Kr	(s)	1.6	0.6	0.3	(o)	1.9	0.9	0.5	
	(p)	6.0	0.6	0.4	(p)	2.5	0.2	0.1	
^{76}Sr	(o)	6.3	4.2	3.3	(o)	6.4	3.7	2.9	
	(p)	12.1	5.2	4.7	(p)	12.2	3.1	3.0	
^{78}Sr	(s)	2.0	1.4	0.9	(s)	2.9	2.2	1.3	
	(p)	9.9	2.7	2.7	(p)	8.3	2.3	2.1	
^{80}Sr	(s)	1.3	0.3	0.2	(s)	2.1	0.6	0.3	
	(p)	7.1	1.0	0.9	(p)	6.3	0.8	0.5	

Table 7. Results from bare $2qp$, TDA, and RPA calculations for the half-lives of Ge, Se, Kr, and Sr isotopes. The results correspond to the two Skyrme forces SG2 and Sk3, as well as for the different shapes oblate (o), prolate (p), or spherical (s), where the minima occur for each isotope. Experimental values are from Ref. [15].

		exp.		2qp	SG2 TDA	RPA		2qp	Sk3 TDA	RPA
^{64}Ge	seconds	63.7	(o)	14	135	176	(o)	12	122	154
			(p)	20	208	259	(p)	19	161	206
^{66}Ge	hours	2.26	(o)	0.3	2.8	3.6	(o)	0.5	3.7	4.9
			(p)	0.2	2.9	4.1	(p)	0.6	4.9	7.2
^{68}Se	seconds	35.5	(o)	18	166	187	(o)	14	132	150
			(p)	12	111	128	(p)	14	105	129
^{70}Se	minutes	41.1	(o)	12	69	90	(o)	22	76	95
			(p)	6	39	44	(p)	19	137	180
^{72}Kr	seconds	17.2	(o)	3	12	15	(o)	3	13	16
			(p)	2	11	13	(p)	3	15	21
^{74}Kr	minutes	11.5	(o)	1.1	7.4	10.0	(o)	0.6	4.1	5.5
			(p)	2.3	10.4	14.0	(p)	3.1	19.4	28.7
^{76}Kr	hours	14.8	(s)	0.9	3.8	6.4	(o)	1.2	2.1	4.3
			(p)	0.2	2.3	3.1	(p)	0.7	12.1	15.0
^{76}Sr	seconds	8.9	(o)	2	9	11	(o)	2	9	11
			(p)	4	28	35	(p)	5	30	38
^{78}Sr	minutes	2.7	(s)	0.8	2.3	3.3	(s)	0.3	0.7	1.1
			(p)	0.6	3.9	5.1	(p)	2.0	9.4	10.9
^{80}Sr	hours	1.8	(s)	0.8	4.4	6.8	(s)	0.7	3.1	5.0
			(p)	0.2	1.4	1.5	(p)	0.4	1.6	2.0

Table 8. Results for various values of the particle-particle coupling strength κ_{pp} [MeV] from RPA calculations with the force SG2. The table contains the summed GT strengths $\sum B_{GT}$ [$g_A^2/4\pi$], the summed strengths up to Q_{EC} , \sum_{EC} [$g_A^2/4\pi$], and the half-lives $T_{1/2}$ [minutes]. The results are for ^{70}Se with prolate and oblate shapes. The experimental half-life is $T_{1/2} = 41.1$ min.

κ_{pp}	$\sum B_{GT}$	prolate		$T_{1/2}$	oblate	
		\sum_{EC}			\sum_{EC}	$T_{1/2}$
0	6.33	1.31		43.9	5.81	0.67
0.02	5.90	1.42		41.5	5.42	0.71
0.05	5.15	1.69		35.3	4.74	0.80





































